

Name .....

School .....



WINCHESTER  
COLLEGE

## Winchester Election

### **Mathematics II**

Monday 2<sup>nd</sup> May 2022

Time allowed: 1 hour 30 minutes

Total marks: 100

**CALCULATORS ARE NOT ALLOWED.**

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.

Diagrams are not drawn to scale.

*Do not be discouraged if you do not finish.*

1.	Evaluate: a) $5 \times 2^2 \times 5^3 \times 2^4$ .	b) $\sqrt{0.36}$ .	[1] [1]
	c) $56 + 98 \times 56 + 56$ .	d) $\frac{27}{7} \div \frac{9}{14}$ .	[2] [2]
	e) $\frac{\sqrt{123} \times \sqrt{123}}{(\sqrt{41})^2}$ .	f) $\sqrt{(2\sqrt{13})^2 - 3}$ .	[2] [2]
	g) $\frac{0.012 \times 0.0125}{0.0000015}$ .	h) $\sqrt{\frac{2\sqrt{64}}{\sqrt[3]{64}}}$ .	[2] [2]

2.

Solve:

a)  $3(2 - x) - 4(5x - 6) = -16$ .

b)  $\frac{1}{3x - 7} = \frac{2}{5x + 9}$ .

[2]  
[2]

c)  $\sqrt{\frac{27}{x}} = 9$ .

d)  $\left(\frac{x - 7}{3}\right)^3 = 125$

[2]  
[2]

e) Expand and simplify  
 $(a - b)(a + b)$ .

f) Expand and simplify  
 $(c - d)(c + d)(c^2 + d^2)$ .

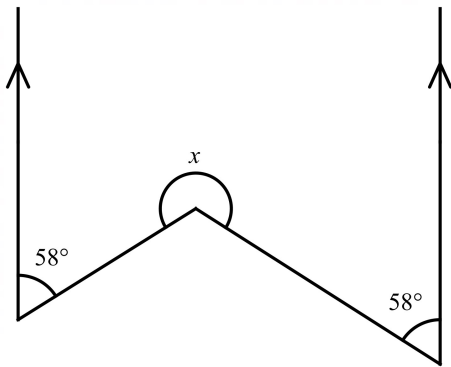
[1]  
[2]

g)  $y$  is currently the subject of the formula,  $y = \sqrt{\frac{a+b}{x-c}}$ . Make  $x$  the subject.

[2]

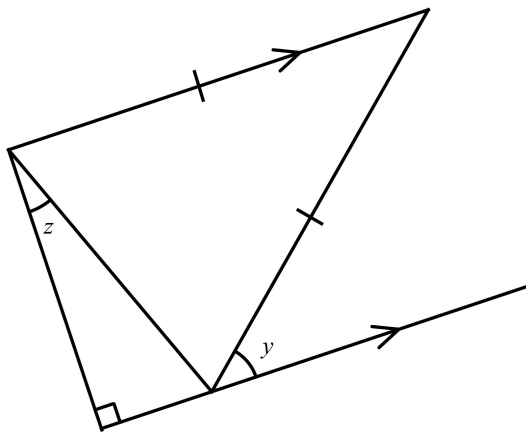
3. a) In the diagram below, there is a pair of parallel lines. Find the angle  $x$ .

[2]



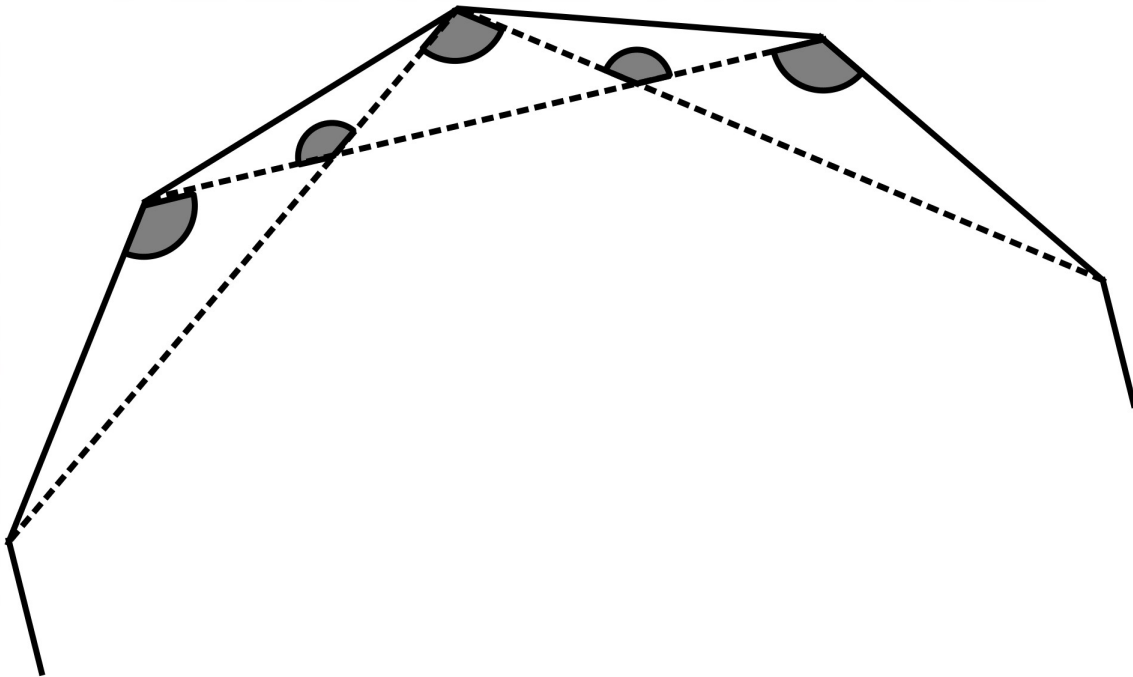
b) In the diagram below, there is a pair of parallel lines. Find an equation linking  $y$  and  $z$  and simplify it.

[3]



c) The solid lines in the diagram below show part of a regular ten-sided shape. Find the sum of the shaded angles.

[4]



4. Kiril is writing a book. He has a rule that Chapter 1 must be 1 page long, Chapter 2 must be 2 pages long, etc. Chapter  $n$  is  $n$  pages long.

a) After six chapters are written, how many pages has Kiril written?

[1]

b) After writing 50 pages Kiril stops for a break, leaving a chapter unfinished. Which chapter was he writing when he stopped?

[1]

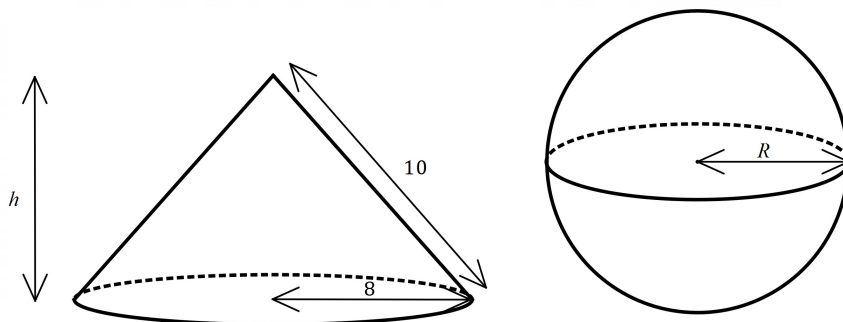
c) Kiril then keeps writing until he has finished the book. He takes it to the publisher, who tells him to add another 110 pages. He adds another 110 pages, still following his rule, and this adds exactly five more chapters to his book. How many chapters has he written now?

[3]

5. The surface area of a cone is equal to  $\pi r(r + l)$ , where  $r$  is the cone's radius and  $l$  is the cone's slant height.  
The surface area of a sphere is equal to  $4\pi R^2$ , where  $R$  is the sphere's radius.

- a) The diagram below shows a cone,  $C_1$ , with radius 8 and height  $h$ . The slant height of the cone is 10. The diagram also shows a sphere with radius  $R$ .  
The surface areas of these two shapes are the same. Find the value of  $R$ .

[3]



- b) Show that the height of the cone,  $h$ , is 6.

[1]

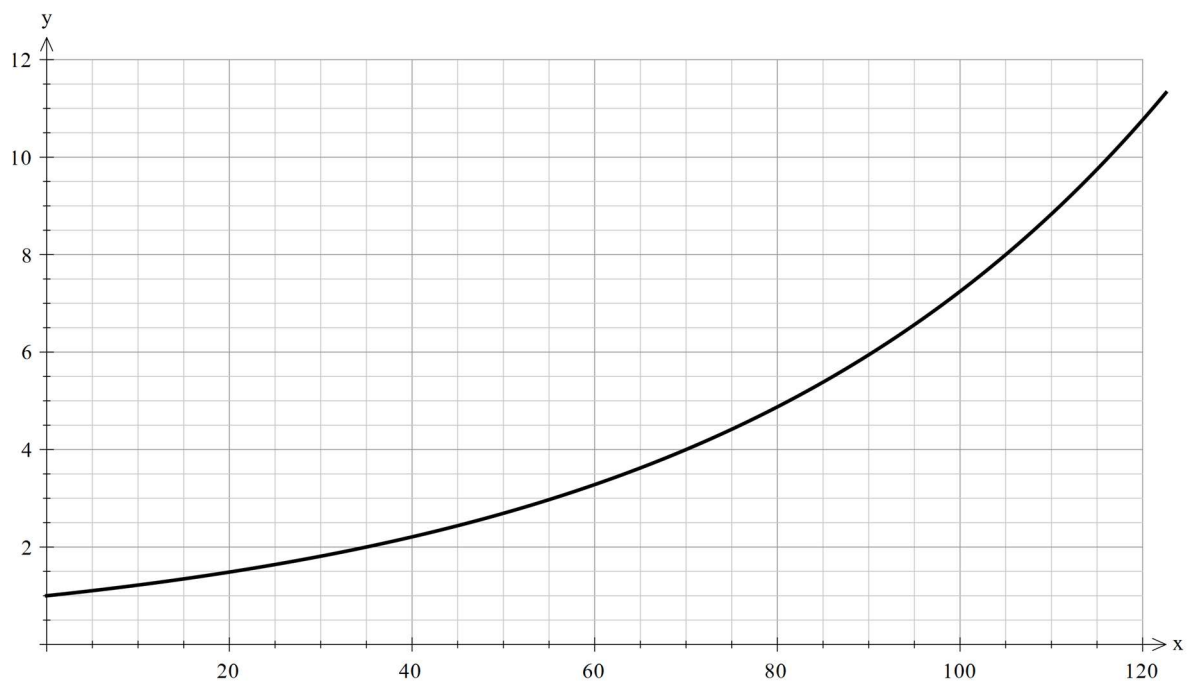
- c) A second cone,  $C_2$ , has height 4 and is mathematically similar to the cone  $C_1$  above.  
What is the surface area of cone  $C_2$ ?

[2]

- d) The volume of cone  $C_1$  is  $128\pi$ . A third cone,  $C_3$ , has height 1.5 and is also mathematically similar to the cone  $C_1$ . What is the volume of cone  $C_3$ ?

[3]

6. Below is the graph of  $y = 1.02^x$  which you may use to help you in the question below.



The mass of Percy the kitten increases by 2% every day. He currently weighs 200g.

a) After roughly how many days will his mass have doubled? [1]

b) How many days until his mass is 2.4kg? [2]

c) If Percy continues to put on mass at the same rate, how long will it take for him to increase from 1kg through to 432kg (the mass of two large lions)? [3]



7. When a number is inputted into a machine, the machine's output is how many distinct prime factors the inputted number has. For example, an input of 12 would have an output of 2, since 12 only has two distinct prime factors, 2 and 3.

a) What is the output when the input is 30?

[1]

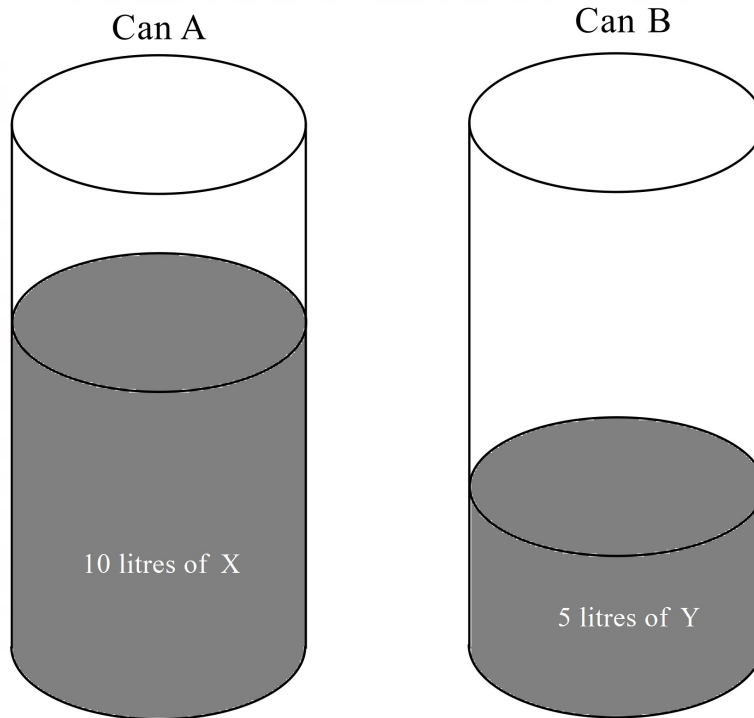
b) What are the three smallest numbers that when inputted into the machine give an output of 4?

[3]

c) Mirabel tells you she inputs two numbers. The inputted numbers multiplied to make 90. The outputted numbers multiply to make 4. What two numbers did she input?

[2]

8. Can A has 10 litres of liquid X. Can B has 5 litres of liquid Y.



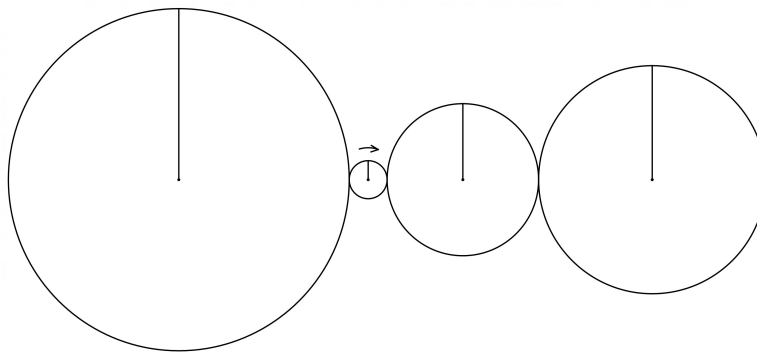
- a) Half the liquid in Can A is poured into Can B. The liquid in Can B is mixed thoroughly and then half of the liquid (5 litres) is poured back into Can A. What is the ratio of liquid X to liquid Y in Can A?

[2]

b) Instead an extra  $n$  litres of liquid X is first added to Can A. Half the liquid in Can A is poured into Can B. The liquid in Can B is mixed and then half is poured back into Can A. The ratio of liquid X to liquid Y in Can A is now 27:5. Find  $n$ .

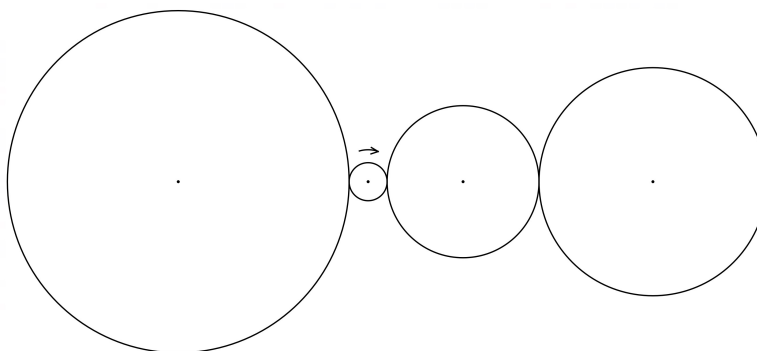
[5]

9. The diagram shows four wheels whose radii are 9m, 1m, 4m and 6m respectively. Each wheel has a line drawn on it, and initially all lines point upwards as shown in the diagram below.
- The smallest wheel makes one full turn clockwise each day.
- The wheels are in contact and do not slip past each other, so as each turns, its neighbours do as well.
- (This means that an ant, sat on the outside edge of any of the wheels would travel at a speed of  $2\pi m$  per day.)*



- a) On the diagram below draw the positions of the lines after three days.

[3]

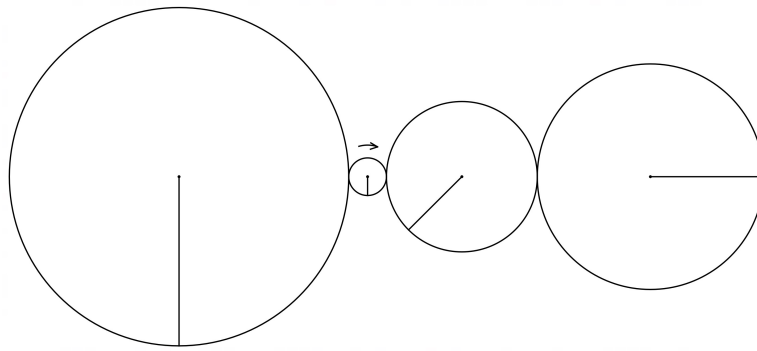


- b) After how many days are all four wheels back at their original position for the first time?

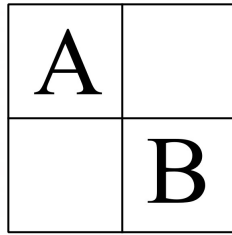
[2]

c) After how many days do all four wheels look like the figure below?

[3]



10. Percy the cat moves either up, down, left or right (never diagonally) within the grid below.



How many ways are there for Percy to move from A to B for the **first time** in:

a) Exactly two moves? [1]

b) Exactly three moves? [1]

c) Exactly four moves? [1]

d) Exactly eight moves? [1]

For the rest of this question please use this new grid below.

A	
	B

How many ways are there for Percy to move from A to B for the **first time** in:

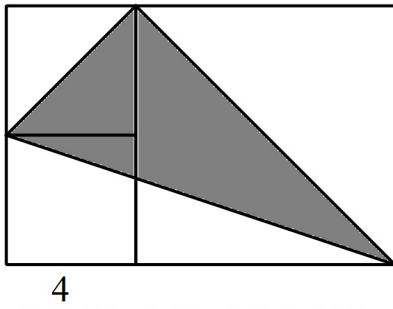
e) Exactly four moves?

[2]

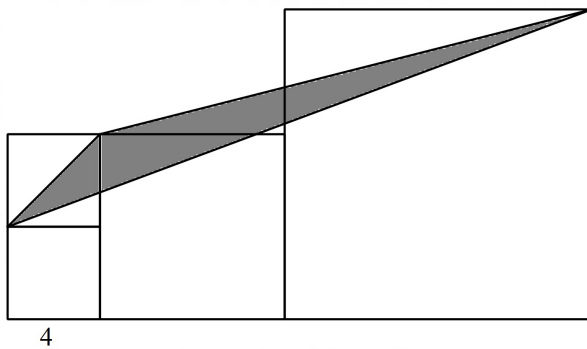
f) Exactly eight moves?

[3]

11. a) Three squares are placed next to each other. The length of one side of the small square is 4. What is the area of the grey triangle? [3]

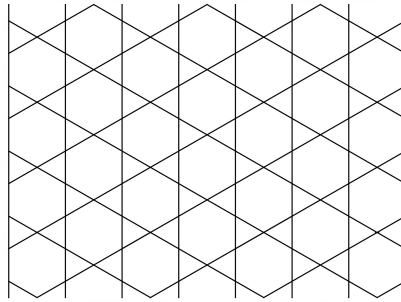


- b) A fourth square of unknown size is added. What is the area of the grey triangle in this diagram? [3]





12. A large floor is tiled with equilateral triangles and regular hexagons as shown below.

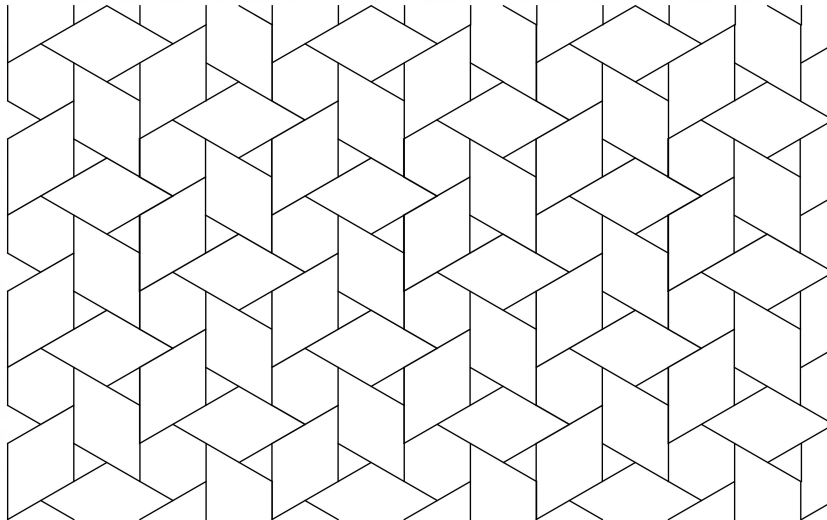


Nikita says that since every hexagon is adjacent to six triangles, the ratio of number of hexagons to number of triangles must be 1:6.

- a) Explain briefly why Nikita is not correct and give the correct ratio of hexagons to triangles. (Assume the floor is large enough that the effect of the edges can be ignored.)

[2]

Another large floor is tiled as shown below. The tiles are equilateral triangles of side length 1, regular hexagons of side length 1 and rhombuses of side length 2.



- b) Find the ratio, number of hexagons: number of triangles: number of rhombuses.

[3]

- c) What fraction of the floor is covered by the triangular tiles?

[3]

